Study on the Mispricing of Index Futures with Stochastic Analysis: Evidence from Chinese CSI300 Index Futures*

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This paper analyzes the pricing of CSI300 stock index futures under a modified Cost-of-Carry framework. We discover a pricing bias in both long-term and short-term contracts which changes consistently over time in both average magnitude and volatility. We adopt a Brownian motion model with drift term to approximate the magnitude and volatility of the bias, and empirically analyze the factors that may affect this pattern. Generally, the bias is affected by market trading volume, investor sentiments, and futures related features. The study provides new evidence for the origin of mispricing in the Chinese stock index futures market and complements conclusions in previous literature.

Keywords: CSI300, stock index futures, Cost-of-Carry, pricing bias

I. INTRODUCTION

The pricing of stock index futures has always been a heated topic in financial researches. The most classic method is known as the Cost-of-Carry model, which is formalized by Cornell and French (1983). This framework suggests theoretically the pricing of stock index futures in a perfect capital market with no frictions. However, as various obstacles can lead to the existence of arbitrage opportunities and imperfect market conditions (Kamara (1988), Klemkosky and Lee (1991), Krishna and Suresh (1985) and many others), this pricing model fails to give robust estimations of the real world. Despite of this ,the Cost-of-Carry model with modified terms is still believed to provide indifferent performance in pricing compared with other frameworks (Brailsford and Cusack (1997), Chow, McAleer and Sequeira (2000), MacKinlay and Ramaswamy (1988), and Marcinkiewicz (2016)). Therefore, it is still widely used as a benchmark in examining the mispricing phenomenon in the index futures market.

Many empirical evidences have proved the underpricing of the cost-of-carry framework in different markets, such as the studies from Gay and Jung (1999), Pope and Yadav (1994), and Lin, Lee and Wang (2013). And there are also discussions on the pricing bias in China (Qiao, Teng, Li and Liu (2019), Wang, Wang, Li and Bai (2019), Zheng and Lin (2015), and Chen (2018)). However, as many of these researches focus on the relation between the spot price and the futures price as well as the magnitude of the bias, few has thrown light on the detailed features of the pricing bias and its futures-based characters. Therefore, the purpose of this study is to investigate the characters of the pricing bias, and provide insights on the origin of mispricing with evidence from China.

In this study, we empirically demonstrate the pricing deviation of the CSI 300 stock index futures under the modified Cost-of-Carry framework. We discover a time-varying and robust pattern in the pricing bias in both long-term and short-term contracts. Generally speaking, the bias shows a trend of consistently changing with more intensive volatility. And it quickly turns to zero at the end of its survive time. We model the magnitude and volatility of the pricing bias with a Brownian motion model, and conduct several GLS regressions to investigate the potential influence of market-based factors on each term in the model.The results show that the intercept term and the linear term are effected by both market factors, investment sentiments, and the individual feature of each futures. The influence varies for long and short contracts. The stochastic term can be well-described with a quadratic polynomial of survive time t.

The study extends the understanding of the relation between market characters and the pattern of the pricing bias, and provides further suggestions on the cause of mispricing in the Chinese stock index market.

The main body of the paper is organized as follows. First, we will go through some of the fundamental literature in stock index pricing. Next, we will provide some descriptive evidence on the pattern of the pricing bias under our framework. Then, we will build a stochastic model to characterize the features mentioned in part 2. We will fit the model with real world data and conduct empirical analysis on the causes of the bias. Finally, this

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paper concludes with a few remarks.

II. LITERATURE REVIEW

A. Literature on Stock Index Futures Pricing Thoery

The pricing model of stock index futures has been developing and diversifying since the 20th century. Most classic forms of stock index pricing models base on the form of the cost-of-carry model, which is developed from the pricing model on commodity futures. A basic form of the stock index futures pricing model is given by Cornell and French (1983) under the following assumptions:

1) Perfect capital markets with no taxes or transaction costs, no restriction on short sales, and perfectly divisible assets.

2) Equal and constant risk-free borrowing and lending rates.

By conducting a no-arbitrage analysis, they give the pricing model in the form of:

$$F(t,T) = P(t)e^{(r-d)(T-t)}, \tag{1}$$

where $F(t,T)$ is the futures price at time t for a contract that matures at time T, $P(t)$ is the price of stock at time t, r is the risk-free rate, and d is the dividend yield.

With similar specifications, Modest (1984) took into consideration the effect of transaction costs, which can be summarized as:

$$\begin{aligned} &\frac{P_t + C_{PL} + C_{FS} - \sum_{t=\tau}^{T} B(t,\tau)d}{B(t,T)} \\ \geq &F(t,T) \\ \geq &\frac{P_t - C_{PS} - C_{FL} - \sum_{t=\tau}^{T} B(t,\tau)d}{B(t,T)}, \end{aligned} \tag{2}$$

where C represents the transaction cost of different options, $B(t,\tau)$ is the price of riskless pure discount bond that pays 1\$ at $T > t$. Similar specifications are also seen in the works of Klemkosky and Lee (1991), where more discussion on the effect of taxes and different hedging opportunities among different investors are included.

Another development of this framework is introduced by Krishna and Suresh (1985), which includes the stochastic process of stock index and interest rates. This model assumes that the dynamic process of the spot price follows:

$$dS(t) = [\alpha(S,t) - \delta S]dt + \sigma_1 S dz_1, \tag{3}$$

where $\alpha(S,t)$ is the cum dividend expected change in the price, and $z_1(t)$ represents a standard Wiener process. The uncertainty of the interest rate is captured by:

$$dr(t) = \kappa(\mu - r)dt + \sigma_2\sqrt{r}dz_2, \tag{4}$$

where μ is the long-term mean of the interest rate. z_2 follows a Wiener process different from the spot price. By assuming $Cov(z_1, z_2) = 0$, the authors give a pricing model of:

$$H(S(t), r(t), t; \tau_2) = S(t)a(\tau_2)exp[b(\tau_2)r(t)], \tag{5}$$

where $\tau_2 = T_2 - t$ is the option's maturity, a and b are functions of τ_2 [1].

Despite the popularity of the cost-of-carry model, there is challenge on whether the set up would ignore the interactions between spot and futures markets (Hemler and Longstaff (1991)). And there is empirical evidence suggesting a systematic pricing deviation from the cost-of-carry model (MacKinlay and Ramaswamy (1988), Kamara (1988), and Hemler (1990)). In the study of Hemler and Longstaff (1991), they developed a closed-form general equilibrium model in a continuous-time production economy characterized by stochastic interest rates and fluctuating level of market uncertainty to characterize the variance of returns on the market. The natural logarithm form of the equilibrium takes the form of:

$$L_{\tau t} = \alpha_\tau + \beta_\tau r_t + \gamma_\tau V_t + \varepsilon_t, \tag{6}$$

where

$$L_{\tau t} = ln(\frac{F_t e^{\rho\tau}}{W_t}),$$

W denotes a representative investor's wealth, τ is the time of maturity, and V is the local variance of stock index return.The specification finds the significant explanatory power of market volatility, which is not considered in the classic cost-of-carry model. To demonstrate the origin of market volatility, Hsu and Wang (2004) discussed the incomplete arbitrage mechanism of the stock index future market, and add the price expectation of underlying assets into the pricing of stock index futures. The solution to their model is Equation (20) in their paper:

$$F(S,t) = (S_t - D_t)e^{\mu_\alpha(T-t)}. \tag{7}$$

Here μ_α is given by:

[1] The result is the solution to a partial differential equation under certain assumptions. For more detail of the derivation see Equation 10 & 12 in Krishna and Suresh (1985) .

$$\mu_\alpha = (\mu - q) - \mu_f \frac{\sigma}{\sigma_f}, \tag{8}$$

where μ and q are the constant expected growth rate in S and the dividend yield respectively. μ_f and σ_f denotes the instantaneous expected return on futures and its standard deviation. A testable form of (8) is:

$$\mu_{\alpha,t-1} = \frac{1}{T-(t-1)} ln(\frac{F_{t-1}}{S_{t-1}}). \tag{9}$$

Although there have been challenges on the cost-of-carry model, many researches conclude that the framework, with some modifications to more realistic market condition, does not differ much from other models in performance (Brailsford and Cusack (1997), Chow, McAleer and Sequeira (2000), and Marcinkiewicz (2016). See Chow, McAleer and Sequeira (2000) for detailed discussions on the comparison). Therefore, we will employ the cost-of-carry model as a baseline framework in this research for simplicity.

B. Literature on Empirical Evidence and the CSI 300 Stock Index Futures

As the cost-of-carry framework remains the most frequently used pricing theory in empirical studies, researchers have been focusing on the testing and explanation of price deviations. Many evidences sugget an underpricing of the cost-of-carry model. Gay and Jung (1999) examined empirical evidence from the Korean stock index futures and emphasize that the influence of restrictions on short sales are potential cause of underpricing, which is supported by evidence from London (Pope and Yadav (1994)), German (Kempf (1998)), Taiwan(Lin, Lee and Wang (2013)), and Hong Kong (Fung and Jiang (1999)). Similar conclusions also apply for CSI 300. By analyzing the 2015 short-selling restrictions on CSI 300 stock index futures, Andrew, Jun, Jin and Jin (2019) found that the restriction reduced futures over-pricing and increased under-pricing situations. These studies all point out the mis-pricing of the cost-of-carry model and the potential factors.

Some studies investigate the CSI 300 stock index futures volatility from other perspectives.Wang, Wang, Li and Bai (2019) reported that a LHAR-RV-CJ prediction model has better prediction on the volatility of CSI 300 stock index futures compared with other time-series analysis method. Qiao, Teng, Li and Liu (2019) demonstrated that the iVX index has notable influence on the volatility of CSI 300 futures with evidence from high frequency data. Many other works reported considerable arbitrage opportunities and price deviations in CSI 300 futures in China (Zheng and Lin (2015), Liu and He (2018), Chen (2018), and Tang and Liu (2020)). According to Zheng and Lin (2015) and Tang and Liu (2020), these market imperfection could be largely explained by investor sentiments. As there are limited discussions on potential factors that may contribute to market imperfection, our research can act as a supplement to the previous literature.

III. DEFINITION OF PRICING BIAS AND DESCRIPTIVE STATISTICS

A. Definition of Pricing Bias

According to the classic model based on no arbitrage (Cornell and French (1983)), the price of stock index futures should follow the equation below:

$$F_t = S_t e^{r(T-t)}, \tag{10}$$

where S_t refers to the stock price at time t, T refers to the mature date of the contract and r is the risk-free interest rate. In classic theory, this equation holds as stock index futures is a special kind of financial product, with no inventory cost and convenient yield. Thus according to the no arbitrage theory, the only term in need of consideration is risk-free interest rate. Based on this model, we define the pricing bias x_t as follows:

$$F_t = S_t e^{(r-x_t)(T-t)}, \tag{11}$$

where x_t is a function of time t and measures the bias between implied discount rate in the stock index futures market and risk-free interest rate. Here we adopt a similar specification as Hsu and Wang (2004), but we divide the μ_α in their equation (9) into two parts. To explicitly show the definition of x_t, from (11), we have:

$$x_t = x(t) = -\frac{\ln F_t - \ln S_t}{T-t} + r. \tag{12}$$

A series of x_t can be calculated using (12) for each contract separately, and this x_t is the target of our analysis. In the following sections, we will refer to x_t as the pricing bias if not specified.

B. Data

To obtain a typical and stable dataset, we obtain the contract data of CSI 300 stock index futures from 2015 to 2020. In this case, we avoid the potential influence from the 2015 stock crush and stock index futures market regulation (Andrew, Jun, Jin and Jin (2019)) as well as the shock from the Covid-19 pandemic. There are 64 contracts in total from 'IF1501' to 'IF2006'. We eliminate some of the contracts in later empirical analysis due to failure in matching these data with the CSI 300 Index.

The prices involved in this research refers to the daily close price. We adopt the yield rate of Chinese 10-year treasury bonds as proxy for the risk-free interest rate. The reason why we do not use the yield rate of treasury bonds with the similar lifespan to the contracts is because treasury bonds have better fluidity, which will render lower yield rate than the real risk-free interest rate. However, according to the term structure of interest rate, interest rate with longer terms is usually higher than its shorter counterparts, which indicates that yield rate of 10-year treasury bonds will compensate for the problems using short-term ones with similar lifespan mentioned above.

Before moving to the summary statistics of our dataset, we first provide a comparison of the trend of the CSI 300 Index and the dominant contract of CSI 300 stock index futures in (7). It is clear to see a high correlation between the CIS 300 spot and its futures, as is proved by many (Qiao, Teng, Li and Liu (2019) and Tang and Liu (2020)). As we eliminate the effect of the spot market in the following sections, the analysis would mainly focus on the parts that could not be explained by the CSI300 index directly.

In the following analysis, we will divide the contracts into two groups: contracts that last longer than 50 days (included) are defined as *Long* contracts, with their counterparts noted as *Short* contracts. The reason for doing this is because there is a clear cut-off of in the duration of contract at around day 45 in CSI 300 futures.

IV. DISCRIPTIVE EVIDENCE

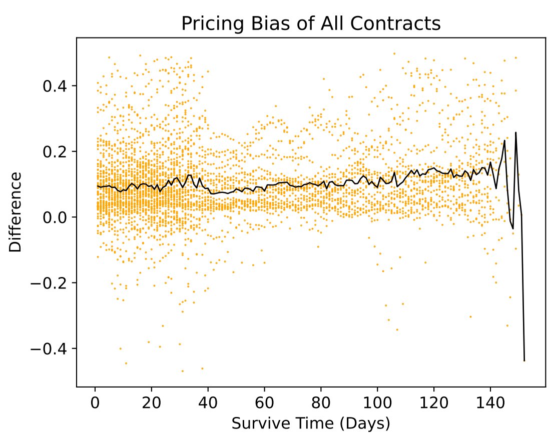

FIG. 1. Pricing Bias of All Contracts, 2014-2020[a b]

[a] *Source:JointQuant Database.*

[b] *Notice:Pricing biases over 0.5 are eliminated as outliers. Both long and short contracts are included in the picture.*

To show some basic facts and characters of the pricing bias, we first give some descriptive evidence of the pricing bias, x_t, mentioned in (12).

Figure 1 is a picture of the relation between the survival time of all the contracts and their daily average pricing bias. Here daily average pricing bias means the average of the pricing bias of all contracts when they reach a same survival day. There are several points worth noticing in this picture. First, the average pricing bias remains positive during the whole survive time for all the contracts. This is highly consistence with studies from Jian, Deng, Luo and Zhu (2018), Zheng and Lin (2015), and Chen (2018), which all indicate a significant price deviation in CSI 300 futures from the Cost-of-Carry pricing. However, it is worth notifying that the pricing deviation defined in our research is different from these works.

Second, the observations can be divided into two period according to the density of the points. The first period is the time when existing days of contracts are shorter than approximately 50 days, and the second period is the time when existing days of contracts are longer than 50 days.

Finally, during the first period, it is clear to see that the pricing bias keeps increasing with much larger volatility. However, this trend disappears quickly when the existing days come near the end of the first period. The same trend also exists during the second period. As is mentioned in the last section, the contracts are divided into two kinds, short-term contracts and long-term contracts. We hypothesize that the two periods reflect the trend of short-term contracts and long-term contracts respectively. If the assumption holds, it means that the trend exists regardless of the length of a contract. This indicates that the phenomenon is systematic in the stock index futures market.

To demonstrate it more explicitly, we plot the trend of short and long contract with the same specification respectively, as is shown in figure 2. The time-varying features described above holds, with the increasing trend significantly different from 0 in both cases.

Figure 3 gives an evidence clear evidence on the time-varying pattern of the volatility of the pricing bias. We normalize the data by subtracting the mean and divide it by its standard deviation for clarity. As is depicted above, short-term contracts show a clear increasing trend in volatility over time. The same result holds for long-term contract. We can also notice a sharp decrease of the volatility during the last few days of the short-term contracts, though the same phenomenon is less distinct in long-contract. The student T test for the short-term contracts' pricing bias proves the significance of our observations ($T = 16.36$). And the same conclusion also applies for long-term contracts ($T = 21.46$).

The student T test value for As the picture is the result of average value with no specific information about each contract, we list the results of some statistical tests below for reference:

This evidence lays a solid foundation for our following analysis and modeling, where we will discuss further on the factors that influences the trend as well as more testings on the phenomenon. In the next section we will

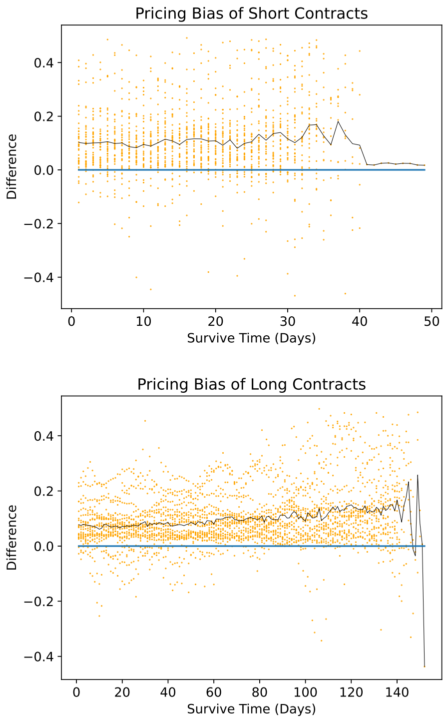

FIG. 2. Pricing Bias of Short-term and Long-term Contracts, 2014-2020[a]

[a] *Source:JointQuant Database.*

introduce our ideas on the modeling.

V. MODELING AND EMPIRICAL METHODS

As we have verified the existence of such trends, we provide a model to quantitatively show this specific motion. The model is constructed based on the Brownian Motion with a drift term:

$$\begin{gathered} dx_{i,t} = dx_i\left(t\right) = \mu_i dt + \sigma_i\left(t\right) dW\left(t\right), \\ \sigma'\left(t\right) > 0, \end{gathered} \tag{13}$$

where $W(t)$ is standard Brownian Motion. μ_i is a constant value for each contract i, measuring the trend of keeping increasing or decreasing. $\sigma(t)$ is an increasing function over t, representing the trend of increasing volatility. Notice that μ_i is different for different contract, as this term might be affected by several factors. We will discuss on the potential factors which could affect term in later sections.

Next, we will give the method for estimating μ_i and $\sigma(t)$. Notice that (13) can be alternatively written in the following form:

$$x_i\left(t\right) = x_i\left(0\right) + \mu_i t + \int_0^t \sigma_i\left(s\right) dW\left(s\right), \tag{14}$$

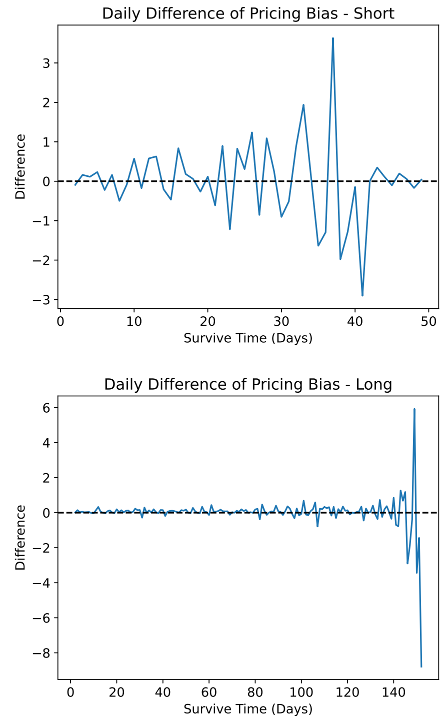

FIG. 3. Daily Difference in Pricing Bias, 2014-2020[a b]

[a] *Source:JointQuant Database.*

[b] *Notice:Daily difference is defined as $x(t) - x(t-1)$, where $x(t)$ is the pricing bias at time t. For clarity, We normalized the data by subtracting the mean and divide it by its standard deviation.*

which is generated by integrating both sides of (13).

A. Estimation of μ_i and $x_i(0)$

Since the model is in the form of a Ito process, we have:

$$\begin{gathered} x_{i,t} \sim N(x(t) + \mu t, \\ \int_0^t \sigma_i\left(x\right)^2 ds), \end{gathered} \tag{15}$$

where $N(\cdot)$ represents a normal distribution. Hence, we can generate the following equation:

$$x_{i,t} = x\left(0\right) + \mu_i t + \varepsilon_t, \tag{16}$$

where ε_t is an increasing function over t. Since the model would be challenged by the heteroscedasticity problem, we would apply the GLS model for the estimation of $x(0)$ and μ_i. And the unbiasedness of μ_i and $x(0)$ would not cause notable challenge to our results.

B. Estimation of $\sigma_i(t)$

According to (15) , we have:

$$x_i\left(t+1\right)-x_i\left(t\right)=\mu_i+\int_t^{t+1}\sigma_i\left(s\right)dW\left(s\right). \tag{17}$$

Notice that the integration term has the following characteristic:

$$\int_t^{t+1}\sigma_i\left(s\right)dW\left(s\right)\sim N\left(0\ \int_t^{t+1}\sigma_i\left(s\right)^2 ds\right). \tag{18}$$

Due to the data limitation, for each contract, we can only observe one process of prices, thus it is impossible to estimate the form of $\sigma(t)$ directly. Alternatively, we will adopt a weaker form:

$$\int_t^{t+1}\sigma_i\left(s\right)^2 ds=\bar{\sigma}_i\left(t'\right)^2\left(t+1-t\right)=\bar{\sigma}_i\left(t'\right)^2$$
$$t\le t'\le t+1,$$

Equation (19) tells us that if we suppose that all the short-term contracts have the same form of $\sigma(t)$, which means:

$$\sigma_i\left(t\right)=\sigma_j\left(t\right)\qquad\forall i\neq j. \tag{19}$$

Thus, we can use the sample variance of all the pricing bias of short-term contracts at time t to estimate the value of $\sigma\left(t\right)^2$. The estimated value is in the form of:

$$\widehat{\sigma\left(t\right)^2}=\frac{\sum_{i=1}^{n}\left(PB_i-\bar{PB}\right)^2}{n-1}, \tag{20}$$

where PB_i is the pricing bias of short-term contract i at time t. $\bar{PB}$ is the average pricing bias of all short-term contracts at time t, and n is the number of short-term contracts. As for long-term contracts, the same method can be used for estimation.

Figure 4 depict the estimated variance over time for both long-term and short-term contracts. There is no convincing evidence that the variance is increasing as time comes close to the maturity of the contract.This indicates that the method inferred above is not appropriate. Actually, the problem might be the supposition: equation (19) may not hold in real financial markets. This indicates that the volatility function is various between different contracts, which might be caused by factors like the macro economic environment, and the investors' sentiment.

To cope with pattern above, we think of a weaker form than the one discussed about above. Although the functions are not the same among all the short-term contracts and long-term ones, it is appropriate to assume that they have a similar variance form with different parameters.

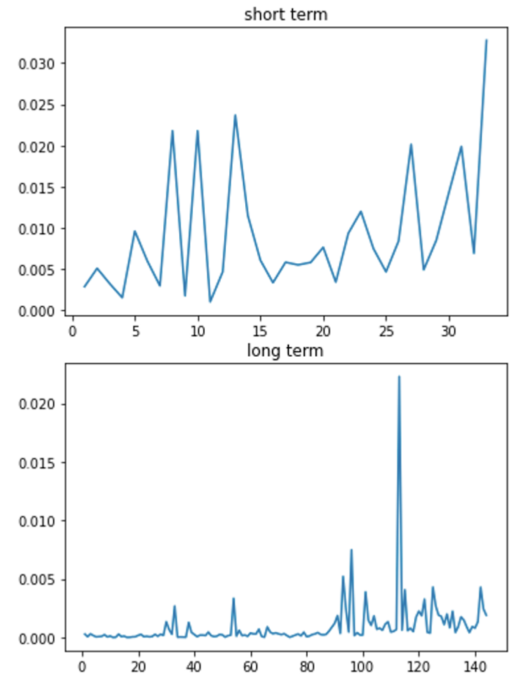

FIG. 4. Estimation of Variance over Time

We suppose $\sigma_i(t)$ follows a linear form:

$$\sigma_i\left(t\right)=\alpha_i+\beta_i t+\varepsilon_i, \tag{21}$$

where $E\left[\varepsilon_i\right]=0$, $Var\left(\varepsilon_i\right)=\sigma_{\varepsilon_i}^2$. Then, we have:

$$x_i\left(t+1\right)-x_i\left(t\right)-\mu_i=\int_t^{t+1}\left(\alpha_i+\beta_i s+\varepsilon_i\right)dW\left(s\right). \tag{22}$$

Due to the characteristic of quadratic variation, we have:

$$E\left[x_i\left(t+1\right)-x_i\left(t\right)-\mu_i\right]^2\approx E\left[\alpha_i+\beta_i t+\varepsilon_i\right]^2, \tag{23}$$

which is derived from:

$$E\left[dI\left(t\right)dI\left(t\right)\right]=E\left[\sigma\left(t\right)^2 dt\right] \tag{24}$$
$$dI\left(t\right)=\sigma\left(t\right)dW\left(t\right).$$

Although we cannot generate the value of $dx_i(t)$, however, compared with the length of the total life of a contract, the time delta of just one day is quite short, which shows the validity for us to using daily difference. Thus, we have:

$$I\left(t\right)=x_i\left(t\right)-\mu_i t-x_i\left(0\right)$$
$$=\int_0^t\sigma_i\left(s\right)dW\left(s\right) \tag{25}$$

$$dI\left(t\right) \approx x_i\left(t+1\right) - x_i\left(t\right) - \mu_i \tag{26}$$

$$\sigma_i\left(t\right)^2 dt \approx \left(\alpha_i + \beta_i t + \varepsilon_i\right)^2 . \tag{27}$$

Equation (23) can be derived by using (25) to (27). We can further the result of (23) by:

$$\begin{aligned} E\left[x_i\left(t+1\right) - x_i\left(t\right) - \mu_i\right]^2 \approx & E[\beta_i^2 t^2 + 2\left(\alpha_i\beta_i + \beta_i\varepsilon_i\right)t \\ & + (\alpha_i^2 + 2\alpha_i\varepsilon_i + \varepsilon_i^2)] \\ E[\beta_i^2 t^2 + 2(\alpha_i\beta_i + \beta_i\varepsilon_i)t + (\alpha_i^2 + 2\alpha_i\varepsilon_i + \varepsilon_i^2)] & \\ = & \alpha_i^2 + \beta_i^2 t^2 + 2\alpha_i\beta_i t. \end{aligned} \tag{28}$$

Equation (28) can be written in another form:

$$y_i\left(t\right) = \left[x_i\left(t+1\right) - x_i\left(t\right) - \mu_i\right]^2 \tag{29}$$
$$= \alpha_i^2 + \beta_i^2 t^2 + 2\alpha_i\beta_i t + \varepsilon_t . \tag{30}$$

As we assume that the difference between the expectation and the real value follows normal distribution, we have:

$$\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right), \tag{31}$$

which indicates that we can simply apply an OLS estimation. Therefore:

$$\begin{aligned} y_i\left(t\right) &= \hat{a} + \hat{b}t^2 + \hat{c}t \\ \widehat{\beta}_i &= \sqrt{\hat{b}} \\ \widehat{\alpha}_i &= \frac{\hat{c}}{2\hat{\beta}} . \end{aligned} \tag{32}$$

Here, the reason that we estimate β_i using $\sqrt{\hat{b}}$ other than $-\sqrt{\hat{b}}$ is that $\sigma_i(t)$ is an increasing function over t. And we can evaluate the accuracy of this estimation using $\hat{a}$. As long as the estimation is quite accurate, the difference between $\widehat{\alpha}_i{}^2$ and $\hat{a}$ should be neglectable.

VI. ESTIMATION RESULTS

A. Estimation value for μ_i and $\sigma_i(t)$

The estimation of μ_i for every contract can be derived from (16). The results are depicted in (5). The detailed values are listed in table VIII. Among all the short-term and long-term contracts, 44 contracts have a significant intercept under the significant level of 10%, and 47 contracts have a μ_i that is significantly different from 0 (results not included). As there are 64 contracts in total, we

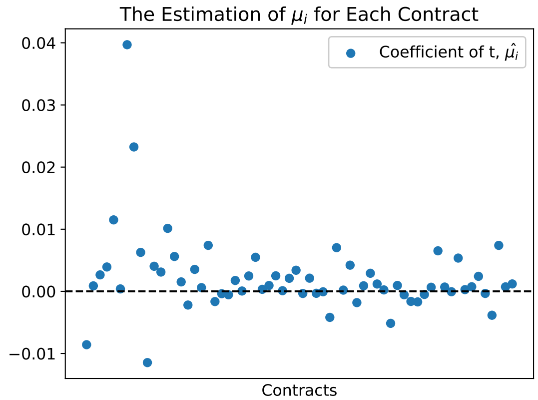

FIG. 5. Estimation of μ_i for All Contracts, 2014-2020[a b]

[a] *Source:JointQuant Database.*
[b] *Notice:Pricing biases over 0.5 are eliminated as outliers. Both long and short contracts are included in the picture. The contracts are ordered by time from left to right.*

can conclude that most of the contracts have the trends discussed in the previous sections.

The estimation of $\sigma_i(t)$ is given by (29), where two features are worth notifying.First, the coefficient of t^2 should be positive. This is quite obvious since the coefficient β_i^2 is in the squared form.

Second, the coefficient of t^2, t, and the constant term should be consistent with the assumption of the model. To verify this, we would first estimate $\hat{\beta_i^2}$, $2\hat{\alpha_i\beta_i}$, and $\hat{\alpha_i^2}$. Then we would conduct a student t test on the following statistic:

$$\hat{\alpha_i^2} - \left(\frac{2\hat{\alpha_i\beta_i}}{2\sqrt{\hat{\beta_i^2}}}\right)^2, \tag{33}$$

which should be not significantly different from 0 if the model is valid.

The estimation and testing results of short contracts are given in table IX. Among all the 42 short-term contracts, 70% of them satisfy this condition first condition, which restricts that the coefficient of t^2 should be positive. By comparing the intercept and $\widehat{\alpha}_i{}^2$ with the student T test ($T = 1.3743$), we verifies that the difference between intercept and $\widehat{\alpha}_i{}^2$ is not significantly different from zero under the significant level of 10%. This indicates that the function form of $\sigma_i(t)$ is appropriate.

Table X shows the results for long-term contracts. In this case, the form of $\sigma_i(t)$ has a better performance than short-term contracts. Over 80% of them satisfy the condition, and student T test also shows that the difference between intercept and $\widehat{\alpha}_i{}^2$ is not significantly different from zero at 10% level ($T = -0.1900$). Notice that although both the short-term contracts and long-term contracts is consistent with the form of $\sigma_i(t)$, our model fits better in long-term contracts than in short-term ones. As is indicated in the results, the student T test statistic has a higher P value than the short-term contracts.

Notice that in both tables we only included estimations with a positive t^2 coefficient. This is because those samples that fail to meet this constrain do not have a well-defined t statistic.

B. Empirical Analysis on μ_i and $x_i(0)$.

In the above sections, we have built up the model of $x_i(t)$, and estimated several parameters as well as function form of $\sigma_i(t)$. However, from the result of estimation, we notice that there actually exist two kinds of contracts with different direction of trend.

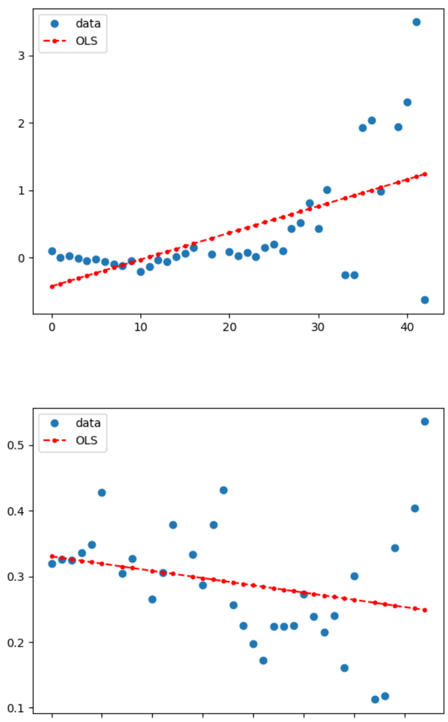

FIG. 6. Pricing Bias of IF1507 and IF1604

Figure 6 provides two examples of contracts. The two short-term contracts have totally different trends. One of them has positive μ_i, and the other has a negative one. Besides, $x_i(0)$ is also quite different among all the short-term contracts and long-term ones. This indicates that μ_i and $x_i(0)$ must be affected by other factors. As CSI300 Stock Index Futures is often invested on the purpose to hedge risks, pricing bias is very likely to reflect traders' sentiment towards the financial markets (Zheng and Lin (2015)). Therefore, we conduct an empirical analysis on the potential factors. The variables are listed in table I.

TABLE I. DEFINITION OF VARIABLES

PARA T	Estimation of μ_i given by 16.
T	Length of the contract duration.
Lng	If it is a long contract($T > 50$).
Ave CSI300	Average volume of CSI 300 during the contract duration.
Ave CSI300 RATE	Average rate of return of CSI 300 during the contract duration.
First Volume	The trading volume of the contract on the first day.
Ave Volume	The average trading volume of the contract.
Ave Money	The average trading money of the contract.
First Money	The trading money of the contract on the first day.
Ave TBill	The average T-bill rate of return within the contract duration.
Num Substitute	Number of other CSI 300 index futures contracts during within the contract duration.
Max Ssub Left	The maximum remaining days of the other short ($T \leq 50$) contracts.
Max Lsub Left	The maximum remaining days of the other long ($T > 50$) contracts.

Equation (II) gives the summary statistics of the variables. Notice that trading volume and money of CSI300 is calculated by weighted average value of its constituent stocks.

To investigate the factors that affect the sign and magnitude of $\hat{\mu}$, we conduct the following estimation strategy:

where $CSI300$ consists of variables that are related with CSI300. $Contract$ includes factors about the characteristic of stock index future contracts. $Substitutes$ denotes the information about other available contracts in the market.

Table III shows the results of this specification. There are several worthy points in this table. First, $AveCSI300Rate$ tends to reducing the changing rate of pricing bias while $AveVolume/10000$ tend to improve the changing rate of the pricing bias. This indicates that if investors invest in the certain contract heatedly, the pricing bias will change more intensively. Besides, similar to $AveVolume/10000$, $AveMoney/10000$ also have such a kind of effect. Other factors, such as $MaxSsubLeft$ and $MaxLsubLeft$ that measures the availability of other short-term and long-term contracts, do not have a significant impaction.

The result above can be explained by the theory that high market trading sentiment trading can result in a fast changing in pricing bias (Andrew, Jun, Jin and Jin (2019)) . As is studied in existing literature, under sev-

TABLE II. Summary Statistics

	N	Mean	SD	Min	Max
PARA T	64	0.00	0.01	-0.01	0.04
T	64	68.08	50.13	7.00	152.00
Long	64	0.33	0.47	0.00	1.00
Ave CSI300 Rate	62	3,634.44	351.87	3,123.91	4,683.41
First Day CSI300	62	3,620.35	408.43	2,649.26	4,786.09
Ave TBill	62	3.34	0.30	2.79	3.85
First TBill	62	3.36	0.33	2.70	4.07
Num Substitute	64	2.91	0.46	0.00	3.00
Max Ssub Left	64	23.52	34.25	0.00	214.00
Max Lsub Left	64	173.66	35.58	0.00	214.00
Ave Volume	64	97,932.95	240,859.72	1,933.66	1,071,229.62
Ave Money	64	1.16e+11	2.90e+11	1969173587.27	1.25e+12

TABLE III. Linear Parameter Analysis: Main Effect[a]

	(1) PARA T	(2) PARA T	(3) PARA T
Long	-0.001	-0.002	-0.002
	(0.001)	(0.003)	(0.003)
Ave CSI300 Rate	-1.323***	-1.318***	-1.334***
	(0.422)	(0.441)	(0.418)
Ave TBill	-0.002	-0.002	-0.002
	(0.002)	(0.002)	(0.002)
Ave Volume/10000	0.002***	0.002***	
	(0.001)	(0.001)	
Max Ssub Left		-0.000	-0.000
		(0.000)	(0.000)
Max Lsub Left		-0.000	-0.000
		(0.000)	(0.000)
Ave Money/10000			0.000***
			(0.000)
Constant	0.008	0.013	0.013
	(0.005)	(0.015)	(0.014)
Observations	62	62	62
R^2	0.48	0.48	0.53

[a] *Note: *** p<0.01, ** p<0.05, * p<0.10. For the clarity of coefficients, we divide Ave Volume by 10000 to normalize the magnitude of this variable.*

TABLE IV. Linear Parameter Analysis: Moderator Effect[a]

	(1) PARA T	(2) PARA T	(3) PARA T
Long	-0.001	-0.010	-0.001*
	(0.001)	(0.008)	(0.001)
Ave Volume/10000	0.002***	0.002***	0.002***
	(0.001)	(0.001)	(0.001)
Ave CSI300 Rate	-1.355***	-1.322***	-1.319***
	(0.441)	(0.425)	(0.425)
Ave TBill	-0.002	-0.003	-0.002
	(0.002)	(0.002)	(0.002)
Long x Ave CSI300 Rate	0.820		
	(0.509)		
Long x Ave TBill		0.003	
		(0.002)	
Long x Ave Volume/10000			0.003***
			(0.001)
Constant	0.008	0.011	0.008
	(0.005)	(0.007)	(0.005)
Observations	62	62	62
R^2	0.48	0.48	0.48

[a] *Note: *** p<0.01, ** p<0.05, * p<0.10. For the clarity of coefficients, we divide Ave Volume by 10000 to normalize the magnitude of this variable.*

eral circumstances, much trading can help reduce the pricing bias, while in other occasions it may worsen the pricing bias. Our results provide another evidence to support this theory. The higher of average trading volume or money always indicates a more heated trading of the contract. Besides, the relationship between CSI300 Index yield rate and changing rate of pricing bias can reflect the time structure of CSI300 Stock Index Futures pricing. During the period with high CSI300 yield return, the pricing bias tend to change more slowly than during the period of low CSI300 yield return.

To investigate the difference between long and short contracts, we generate the cross product of *Long* and other factors. The test on moderator effects is shown in table IV.

As is shown in table IV, average trading volume of long-term contracts have more influence on the changing of pricing bias than short-term contracts. This is partly because long-term contracts indicate higher uncertainty than short-term contracts, thus trading sentiment have a more intense impaction. The moderator effect is significant at 1% level, which indicates a strong signal of heterogenous effects in two types of contracts.

We then conduct an empirical analysis on the factors affecting $x_i(0)$. Table V shows the main regression results. The trading volume of the contract on the first day and yield rate of CSI300 during the past 5 days have

TABLE V. Intercept Analysis: Main Effect[a]

	(1) Intercept x(0)	(2) Intercept x(0)	(3) Intercept x(0)	(4) Intercept x(0)	(5) Intercept x(0)
First Day Contract Volume/10000	-0.334***	-0.321***	-0.328***	-0.339***	-0.343***
	(0.092)	(0.094)	(0.093)	(0.092)	(0.094)
First Day CSI300 Volume/10000	1.728	1.688	1.973	1.728	1.735
	(1.565)	(1.566)	(1.525)	(1.552)	(1.582)
CSI300 Return in 5 Day	-2.114**	-2.147**	-2.193**	-2.150**	-2.152**
	(1.006)	(1.013)	(1.003)	(0.986)	(0.974)
CSI300 Rolling Ave Std in 5 Days	4.074***	3.792***	3.919***	3.936***	3.902***
	(0.936)	(0.940)	(0.890)	(0.879)	(0.854)
Long	-0.037	-0.035	-0.037	-0.034	0.017
	(0.026)	(0.026)	(0.025)	(0.025)	(0.029)
Ave TBill		-0.056			
		(0.043)			
First Tbill			-0.075**		
			(0.035)		
Max Ssub Left				-0.001*	
				(0.000)	
Max Lsub Left					0.001***
					(0.000)
Constant	0.015	0.208	0.266**	0.036	-0.231**
	(0.031)	(0.155)	(0.122)	(0.030)	(0.101)
Observations	60	60	60	60	60
R^2	0.57	0.58	0.59	0.59	0.60

[a] *Note: *** p<0.01, ** p<0.05, * p<0.10. For the clarity of coefficients, we divide First Day Contract Volume and First Day CSI300 Volume by 10000 to normalize the magnitude of this variable.*

a negative effect on the intercept, and volatility of CSI300 yield rate has a positive effect. As for other variables, interest rate of 10-year treasury bond on the first trading day of the contract, max days left of all the short-term contracts coexisting with the target contract have a negative effect on the intercept, and max days left of all the long-term contracts have a positive influence.

The pattern explains explicitly that heated tradings on the first trading day of the contract can systematically reduce the pricing bias. A possible explanation is that for CSI300 Stock Index Futures, much trading on the first trading day contributes to the revealing of true market value, thus triggering the decrease of pricing bias. The volatility of CSI300 shows the extent of market panic. The pattern shows that if people feel more panic, the pricing bias will be higher. This is reasonable as under the more panic environment, investors tend to be less rational and the pricing bias is usually higher. As for CSI300 yield rate, this reflects the time structure of $x_i(0)$. During the period when the financial market has experienced a high yield rate, the pricing bias is systemically lower than other periods. The coefficient of the 10-year treasury bond interest rate indicates that if the risk-free interest rate is higher, the investors can get a higher risk-free return, and more people will invest in the risk-free bonds. Just as the analysis on the effect of volatility of CSI300 yield rate, when there is less panic, the pricing bias is lower. Finally, the coefficients of max days left of other short-term contracts and long-term contracts show that in the short run, the availability of other contracts will help reduce down the pricing bias, but this trend is reverse in the long run.

Similarly, we also analyze if the length of the contracts can adjust the effect of several variables. The results are demonstrated in table VI. As is depicted in the table, short-term contracts are more likely to be influenced by the trading volume of CSI300 on the first trading day, but long-term contracts do not show such pattern. Besides, short-term contracts are more affected by the volatility of CSI300 yield rate than the long-term ones. This is also reasonable as panic always has a more obvious impaction during the short run than in the long run. The results prove that moderator effect applies in both the time-varying character of the pricing bias and the constant part of the bias.

VII. CONCLUSION

In this study, we empirically demonstrate the pricing deviation of the CSI 300 stock index futures under the modified Cost-of-Carry framework. We discover a time-varying and robust pattern in the pricing bias in both long-term and short-term contracts. Generally speaking,

TABLE VI. Intercept Analysis: Moderator Effect[a]

	(1) Intercept x(0)	(2) Intercept x(0)	(3) Intercept x(0)
First Day Contract Volume/10000	-0.391***	-0.322***	-0.355***
	(0.100)	(0.098)	(0.090)
Long	0.107*	0.005	0.138***
	(0.053)	(0.028)	(0.051)
First Day CSI300 Volume/10000	6.700*	1.352	1.870
	(3.393)	(1.605)	(1.631)
CSI300 Return in 5 Day	-2.264**	-2.283**	-2.139**
	(0.926)	(0.971)	(0.858)
CSI300 Rolling Ave Std in 5 Days	3.521***	3.987***	5.375***
	(0.742)	(0.836)	(1.299)
Max Lsub Left	0.001**	0.001***	0.001**
	(0.000)	(0.000)	(0.000)
Long x CSI300 Volume/10000	-6.445**		
	(3.143)		
Long x 5 Day Ave CSI300 Return		1.825	
		(1.468)	
Long x 5 Day CSI300 Rolling Ave Std			-4.577***
			(1.654)
Constant	-0.275**	-0.232**	-0.262**
	(0.111)	(0.101)	(0.103)
Observations	60	60	60
R^2	0.63	0.61	0.64

[a] *Note: *** p<0.01, ** p<0.05, * p<0.10. For the clarity of coefficients, we divide First Day Contract Volume and First Day CSI300 Volume by 10000 to normalize the magnitude of this variable.*

the bias shows a trend of continue and consistent changing with more intensive volatility. And it quickly turns to zero in the last few days of its survive time.

Based on these findings, we develop a model to depict the motion of pricing bias before the last few days of each contract, the main form of which is given by:

$$x_i\left(t\right) = x_i\left(0\right) + \mu_i t + \int_0^t \sigma_i\left(s\right) dW\left(s\right).$$

We then estimate the variance term with a parametric model, and conduct several GLS regressions to investigate the potential factors that may affect other terms in $x_i(t)$. The results show that the intercept term and the linear term are effected by both market factors, investment sentiments, and the individual feature of each futures. The influence varies for long and short contracts. The stochastic term can be well-described with a quadratic polynomial of survive time t. Our model is highly consistent with the real world data, which proves its validity and contributions in describing the market pattern.

There are much to extend in our research. First, due to the limitation of data, we could not verify if the same pattern occurs in other more developed stock index futures markets (for example, the S&P 500 market). It is very likely that the pricing bias we define in this study would behave differently in other markets. Second, we do not explain the origin of the pricing bias in CSI 300 futures. Although there are researches trying to explain it in the aspect of arbitrage restrictions and transaction costs (such as Liu and He (2018),Pope and Yadav (1994), and Fung and Jiang (1999)), many have reported that a large part of the bias remains unexplained. The patterns described in this research may have marginal contributions on the discovery of the source of price deviation. Finally, since the CSI300 stock index futures market is a relevantly young and under-developed market compared with the futures market in developed countries, it remains to be seen if the conclusions of this research still holds as time goes on. We would watch the changes in the market closely and update our results in future studies when necessary.

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Appendix A

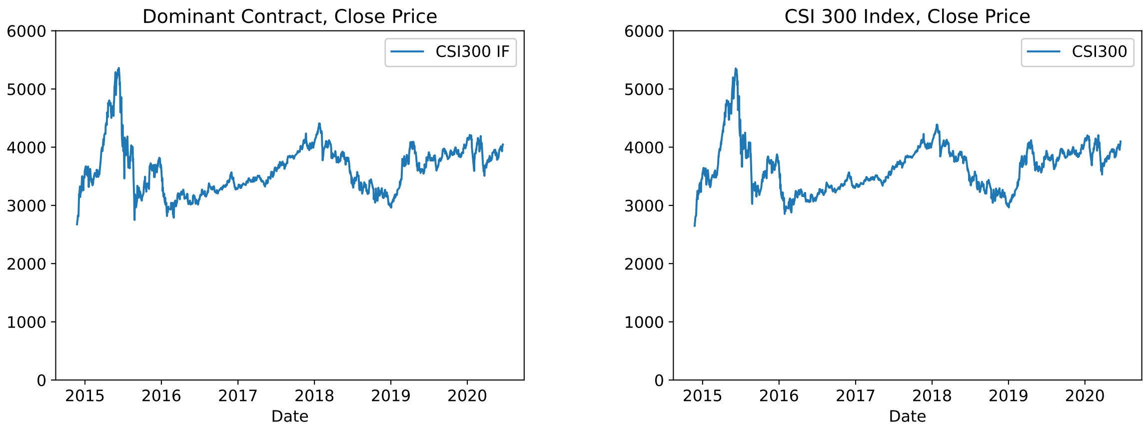

FIG. 7. CSI 300 and CSI 300 futures Dominant Contract, 2014-2020[a]

[a] *Source:JointQuant Database.*

TABLE VII. Contract Specifications for the CSI 300 Stock Index Futures Contract[a]

Character	Specification
Underlying Contract	CSI 300 Index
Contract Multiplier	CNY 300 / Point
Unit	Index Point
Minimum Size	0.2 Points
Contract Months	Monthly: current month, next month, next two calendar quarters (four contracts in total)
Trading Hours	9:30 a.m. - 11.30 a.m. 13:00 p.m. - 15:00 p.m.
Limit up & down	±10% of settlement price on the previous trading day
Minimum Trading Margin	8% of the contract value
Last Trading Day	The third Friday of the expiration month of the contract. Postponed in case of national holidays
Delivery Day	Third Friday, same as the "Last Trading Day"
Settlement Method	Cash settlement

[a] *See http://www.cffex.com.cn/hs300/ for more information about the CSI 300 Stock Index Futures.*

TABLE VIII. Estimation of μ_i

	Constant	p-Constant	$\hat{\mu_i}$	p-$\hat{\mu_i}$
bias_IF1501	-0.03033	0.808498	-0.00859	0.12473
bias_IF1502	-0.07894	0.062683	0.000882	0.631371
bias_IF1503	-0.38272	0.008171	0.002652	0.019298
bias_IF1504	-0.07014	0.218095	0.003921	0.185459
bias_IF1505	-0.12017	0.135543	0.011499	0.005556
bias_IF1506	-0.06001	0.049956	0.000393	0.175991
bias_IF1507	-0.42673	0.050751	0.039691	6.38E-05
bias_IF1508	0.054776	0.620913	0.023239	8.28E-06
bias_IF1509	-0.27828	6.53E-05	0.006268	3.7E-15
bias_IF1510	0.970335	9.79E-08	-0.01147	0.139871
bias_IF1511	0.354394	6.96E-07	0.004032	0.146281
bias_IF1512	-0.01337	0.491301	0.003104	6.42E-32
bias_IF1601	0.222521	0.003534	0.010136	0.003565
bias_IF1602	0.250621	6.69E-08	0.005609	0.002894
bias_IF1603	0.156314	4.37E-16	0.00152	2.09E-13
bias_IF1604	0.330593	8.2E-12	-0.0022	0.141454
bias_IF1605	0.149031	4.34E-07	0.003531	0.001684
bias_IF1606	0.191459	6.94E-29	0.000599	5.68E-05
bias_IF1607	0.162253	3.19E-11	0.0074	1.63E-10
bias_IF1608	0.211768	4.05E-15	-0.00165	0.019229
bias_IF1609	0.230288	2.22E-42	-0.00038	0.002702
bias_IF1610	0.154901	4.67E-10	-0.00055	0.490682
bias_IF1611	0.115261	3.83E-05	0.001749	0.134893
bias_IF1612	0.140742	1.1E-19	5.43E-05	0.699555
bias_IF1701	0.084083	0.001002	0.002475	0.015662
bias_IF1702	0.074542	0.0401	0.005483	0.001913
bias_IF1703	0.09887	3.68E-28	0.000321	5.49E-05
bias_IF1704	0.099453	1.19E-07	0.00094	0.151118
bias_IF1705	0.083858	0.002437	0.0025	0.034738
bias_IF1706	0.104555	2.92E-42	0.000103	0.081889
bias_IF1707	0.093642	1.36E-05	0.002099	0.011243
bias_IF1708	0.061553	0.000894	0.003394	2.8E-05
bias_IF1709	0.120981	2.24E-52	-0.00034	2.63E-09
bias_IF1710	0.034069	0.396103	0.002117	0.250753
bias_IF1711	0.057942	5.82E-05	-0.00032	0.584331
bias_IF1712	0.083796	1.09E-20	-7.1E-05	0.38519
bias_IF1801	0.098479	0.000196	-0.00419	0.000208
bias_IF1802	-0.0684	0.115764	0.007029	0.00038
bias_IF1803	0.044822	1.64E-06	0.000198	0.046146
bias_IF1804	0.053811	0.040579	0.00421	0.001358
bias_IF1805	0.127782	2.14E-09	-0.00182	0.011981
bias_IF1806	0.003824	0.584216	0.000894	3.2E-22
bias_IF1807	0.11856	0.000191	0.002907	0.017296
bias_IF1808	0.130906	6.34E-07	0.001182	0.206612
bias_IF1809	0.061929	1.43E-12	0.000232	0.006413
bias_IF1810	0.136375	4.2E-07	-0.00515	1.41E-05
bias_IF1811	0.018146	0.201271	0.000949	0.214573
bias_IF1812	0.104049	2.16E-30	-0.00055	1.01E-11
bias_IF1901	0.051997	0.001217	-0.00162	0.015113
bias_IF1902	0.039548	0.052112	-0.0017	0.127616
bias_IF1903	0.072925	3.17E-20	-0.0005	5.52E-10
bias_IF1904	-0.00166	0.93861	0.000655	0.466899
bias_IF1905	-0.0607	0.156951	0.006524	0.001152
bias_IF1906	-0.00149	0.856625	0.000691	1.33E-12
bias_IF1907	0.135174	6.97E-08	-6E-05	0.944109
bias_IF1908	0.021806	0.485986	0.005353	0.000754
bias_IF1909	0.036268	4.29E-14	0.000276	3.26E-08
bias_IF1910	0.050124	0.007526	0.000741	0.368389
bias_IF1911	0.026209	0.306967	0.002413	0.078541
bias_IF1912	0.071131	5.91E-34	-0.00034	1.18E-11
bias_IF2001	0.071206	0.007181	-0.00385	0.001494
bias_IF2002	-0.059	0.280513	0.007394	0.007279
bias_IF2003	0.001358	0.942735	0.000722	0.000778
bias_IF2006	-0.0077	0.283743	0.001192	1.81E-30

TABLE IX. Estimation of the coefficients of $\sigma_i(t)$ for Short Contracts.

	IF1501	IF1502	IF1504	IF1505	IF1507	IF1511	IF1601	IF1711
β	0.0241	0.0035	0.0161	0.0074	0.0212	0.0056	0.0049	0.0020
α	(0.2936)	0.0518	(0.1651)	(0.0795)	(0.2829)	(0.0438)	(0.0267)	(0.0092)
α^2	0.0862	0.0027	0.0272	0.0063	0.0800	0.0019	0.0007	0.0001
BIAS	0.0324	0.0032	0.0063	(0.0027)	0.0173	(0.0039)	(0.0021)	(0.0003)
	IF1801	IF1802	IF1805	IF1807	IF1808	IF1810	IF1901	IF1911
β	0.0017	0.0014	0.0020	0.0018	0.0024	0.0034	0.0041	0.0026
α	(0.0224)	(0.0152)	(0.0575)	0.0294	(0.0259)	(0.0377)	(0.0533)	(0.0184)
α^2	0.0005	0.0002	0.0033	0.0009	0.0007	0.0014	0.0028	0.0003
BIAS	(0.0004)	(0.0003)	(0.0010)	0.0018	(0.0011)	0.0000	0.0007	(0.0003)
	IF1602	IF1607	IF1608	IF1610	IF1611	IF1701	IF1707	IF1910
β	0.0041	0.0040	0.0024	0.0036	0.0040	0.0020	0.0025	0.0045
α	0.0062	(0.0357)	(0.0086)	(0.0402)	(0.0501)	(0.0259)	(0.0346)	(0.0521)
α^2	0.0000	0.0013	0.0001	0.0016	0.0025	0.0007	0.0012	0.0027
BIAS	0.0001	(0.0002)	(0.0002)	(0.0001)	0.0000	(0.0001)	0.0000	0.0007
	IF1902	IF1904	IF1905	IF1907	IF1908			
β	0.0056	0.0036	0.0032	0.0029	0.0047			
α	(0.0689)	(0.0523)	(0.0309)	(0.0388)	(0.0624)			
α^2	0.0047	0.0027	0.0010	0.0015	0.0039			
BIAS	0.0007	(0.0006)	(0.0002)	0.0002	0.0013			
T	1.3743							

TABLE X. Estimation of the coefficients of $\sigma_i(t)$ for Long Contracts.

	IF1506	IF1509	IF1603	IF1609	IF1612	IF1703	IF1706	IF1709	IF1712
β	0.001032245	0.001175	0.000334	0.000214	0.000116	0.000356	0.000251	0.000113	0.000198
α	-0.074938681	-0.02297	-0.04182	-0.01566	-0.00286	-0.01945	-0.01035	-0.00253	-0.00864
α^2	0.005615806	0.000527	0.001749	0.000245	8.15E-06	0.000378	0.000107	6.39E-06	7.46E-05
BIAS	6.55555E-05	0.000592	-0.00096	-0.00018	-7.6E-05	7.06E-05	-6E-06	-3.7E-05	-1.7E-06
	IF1803	IF1806	IF1809	IF1812	IF1903	IF1906	IF1909	IF1912	IF2003
β	0.000342285	0.000237	0.000259	0.000161	0.000222	0.000188	0.000243	0.000202	0.000655
α	-0.01778363	-0.00209	-0.00827	-0.00242	-0.01059	-0.00107	-0.01265	-0.00874	-0.03027
α^2	0.000316257	4.37E-06	6.84E-05	5.86E-06	0.000112	1.15E-06	0.00016	7.64E-05	0.000916
BIAS	7.19338E-05	6.5E-06	-6.2E-05	-3.5E-05	-9.5E-06	4.33E-06	-1.1E-05	4.02E-06	0.000326
T	-0.189953978								