# The Solow Growth Model with an Aging Population

Pusheng Liu[1]

[1]Basis International School, Guangzhou, China,Pusheng.Liu14175-bigz@basischina.com

**Abstract**

The world is aging. This study attempts to add to the literature that describes models for the analysis of aging populations. It will describe modifications on a rather basic model – the Solow Growth Model, making an attempt in a rather new field of economic growth theory. It first defines the traditional Solow Growth Model and its main ideas. Then, it develops an augmented version of the Solow Growth Model that takes in the composition of the population in the analysis as inputs and predicts capital per worker, which is directly related to growth rates. The augmented model is able to predict different growth rates in populations that have different young-old proportions and suggests a more precise way of capturing the savings rate in a population than the original model. Specifically, the savings rate now changes with demographic statistics, which is also more relevant to our use in an aging society.

**Keywords: Solow Growth Model, Aging population, Augmented Solow Model**

# 1 Introduction

This study attempts to develop an augmented version of the Solow Growth Model ( Solow, 1956 ) such that it is useful in the analysis of aging populations. In traditional growth theory, most models assume that population growth in the target economy is positive, which is not the case in many countries in today's world ( Marešová, Mohelská, and Kuča, 2015 ). Few countries in East Asia have suffered from an aging population for over a decade now, and so will many more in the foreseeable future. Thus, being able to analyze economic growth becomes crucial, and the basic Solow Growth Model could constitute a neat beginning.

In the literature that suggests a modification to the Solow Growth Model, a few are related to this analysis. Sasaki et al. (2019) suggested a modified version to account for scenarios with a declining population. He assumed the productivity function to take the constant-elasticity-substitution (CES) form and concluded: "The per capita output growth rate is zero if the technological progress rate is zero and the elasticity of substitution between capital and labor is less than unity." As Sasaki, we will assume that technology is an exogenous variable and focus more on modifying the way the savings rate in an aging population is derived.

Li and Zhang (2015) suggested that the aging population may not necessarily result in unfavorable growth as savings (investment) do not necessarily have a negative correlation with the age of the population. However, their accounting methods for savings rates in an aging population are disagreed by many other scholars. For example, Ong (2022) argued that immediately when a population starts aging, its savings rate will decrease, and Leibfritz et al. (1995) believes the aging population will have an adverse effect on the savings rate. Nevertheless, Li and Zhang's methodology is essential: they assumed in their analysis that the population is simply divided into young and old people, which we will continue to assume.

This suggests that literature specifically on the relationship between an aging population and savings rate is especially important. Disney (1996) suggested an association relationship between aging and savings rate. There is also stronger evidence: a literature review done by Nagarajan, Teixeira, and Silva (2016) not only points out the dominating effect the "consumption and savings" mechanism has as opposed to the other two mechanisms but also concludes a significant and negative relationship between aging and savings and thus growth. More impressively, they argue the scope of their results covers developed, developing, and less developed countries. Pascual-Saez, Cantarero-Prieto, and Manso (2020) , using data from Europe, also suggest that aggregate savings have declined as a result of the aging population. The studies also argue that changes in consumption behavior and involvement of the public sector are important variables in the causal link.

The rest of the paper is organized as follows: section two will briefly introduce the main ideas of the Solow Growth Model, section three will present a modification of the Solow Model to account for changes in an aging population, section four will test the augmented model against economic growth data from China, Japan, and Korea, and section five will conclude the paper also suggest future study topics.

# 2 The Solow Growth Model

This section will develop the core concepts and underlying assumptions that shape the Solow Growth Model. The Solow Growth Model, developed by Nobel laureate Robert Solow in 1956, adds a new dimension to previous economic models by considering the roles of capital stock, savings rates, population growth, and technological progress in driving economic growth.

## 2.1 Basic Assumptions:

1. The Cobb-Douglas production function will be employed in our analysis ( Cobb and Douglas, 1928) . The production function accounts for both capital and labor inputs, and it follows the principle of constant returns to scale. This means that if both capital and labor are increased by a factor, the total output will increase by the same factor.

2. The model assumes a closed economy, disregarding international trade by excluding imports and exports from consideration.

3. Government-related aspects are not part of the model, meaning no government spending or taxation is considered.

4. Each person saves a portion of their income ($s$) and consumes the remainder ($c$).

5. A fraction of the capital ($\delta$) depreciates each year due to wear and tear.

6. The labor force expands at a rate equivalent to the population growth rate ($n$).

## 2.2 The Simplified Production Function:

The Cobb-Douglas production function expresses the relationship between output (Y), capital (K), and labor (L) while considering the total factor productivity (A), which we assume to be the technology level. The constant returns to scale assumption implies that the proportional increase in both capital and labor results in a proportionally equivalent increase in output.

$$Y(K, L) = AL^{\beta}K^{\alpha} \tag{1}$$

This equation underscores that output depends on the combination of capital and labor, driven by the technology level. With constant returns to scale, the output per worker ($y$) can be derived by dividing $Y$ by $L^{\beta}$. We can then take the logarithm with respect to $K^{\alpha}$ so that we have a linear function. These simplifications are without loss of generality for our purposes and allow us to focus on the relationship between output per worker and capital per worker, effectively isolating the effect of capital on economic growth.

$$y = f(k) \tag{2}$$

Here, y represents output per worker, and k represents capital per worker. This equation signifies that output per worker is solely determined by the amount of capital per worker, highlighting the pivotal role of capital in driving economic growth.

## 2.3 The Two-variable Consumption Function:

The model assumes that individuals either consume or invest their income. This forms the basis for the equation relating output (y) to consumption (c) and investment (i):

$$y = c + i \tag{3}$$

Assuming a savings rate (s), the consumption (c) can be expressed as a fraction of the output:

$$c = (1 - s)\, y \tag{4}$$

Consequently, the equations for consumption and investment become:

$$c = (1 - s)\, y \tag{5}$$

$$i = sy \tag{6}$$

This emphasizes the equal importance of savings and investment, with the savings rate playing a crucial role in capital formation and, by extension, long-term economic growth.

## 2.4 The Steady State, Capital Stock, and Growth:

Capital stock significantly influences worker productivity and, consequently, economic growth. This stock is shaped by two key factors: investments and depreciation. By substituting the production function into the investment equation, investment could be written as:

$$i = sf(k) \tag{7}$$

Including depreciation, the change in capital stock per worker (Δk) can be expressed as:

$$\Delta k = sf(k) - \delta k \tag{8}$$

This equation reveals the dynamic interplay between investments, depreciation, and the accumulation of capital stock. It lays the foundation for the concept of the steady state, where capital accumulation through investment is balanced by capital loss through depreciation. Now the steady-state capital $k^*$ may be defined. At $k^*$ , $\Delta k = 0$.

The steady-state represents the long-term equilibrium of an economy, indicating that an economy that has diverged from its steady state will constantly move back toward it. Figure 1 illustrates that if investment surpasses depreciation, capital stock increases until equilibrium is reached, and vice versa. To be more specific, once the capital per worker exceeds the equilibrium $k^*$ , depreciation will be greater than investment, resulting in a decrease in capital stock. On the other hand, if the capital per worker is less than the equilibrium, the overall investment is higher than the overall depreciation, indicating an aggregate growth in capital, thus capital stock will grow until investment is equal to depreciation, where we have $k^*$ .

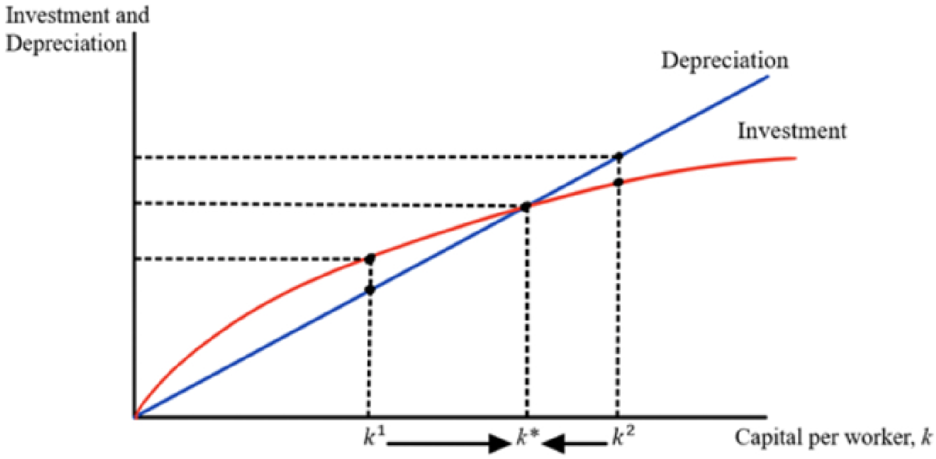


Figure 1: Investment and depreciation ( Huang, 2021)

## 2.5 The Importance of Savings:

Figure 2 highlights the connection between savings rates, investment, and steady-state capital stock. A higher savings rate translates to increased investment, leading to higher capital stock in the steady state.

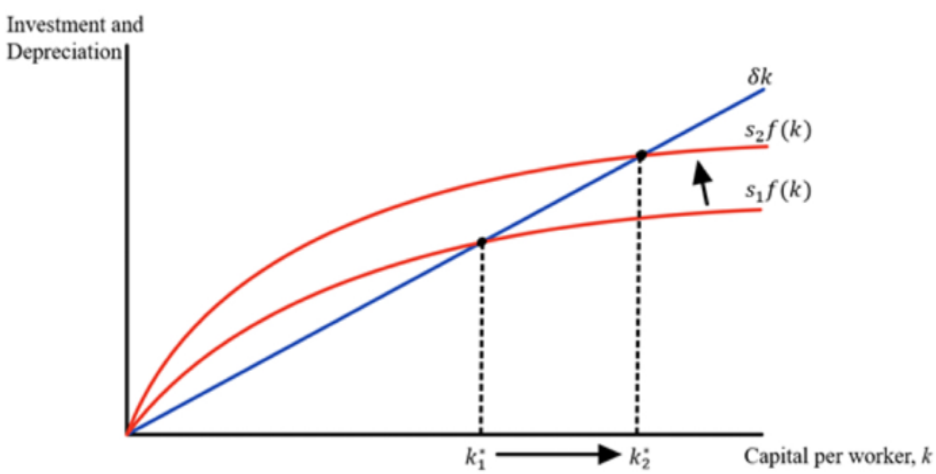


Figure 2: Change in savings ( Huang, 2021)

## 2.6 The Steady State Considering Population Growth:

Population growth is another influential factor affecting capital per worker. As the population expands, other factors being constant, capital per worker decreases. Including the population growth rate ($n$) in the equation, the following could be concluded:

$$\Delta k = sf(k) - (\delta + n) \tag{9}$$

This equation acknowledges that population growth impacts capital accumulation. The concept of break-even investment emerges, representing the investment required to maintain a constant level of capital per worker (in the steady state) while accounting for both depreciation and the needs of new workers.

## 2.7 Consumption under the Solow Growth Model:

The steady-state output ($c^*$) and investment $(\delta + n)k^*$ enable us to calculate steady-state consumption:

$$c^* = f(k^*) - (\delta + n)\,k^* \tag{10}$$

To maximize consumption, we can use the marginal product of capital (MPK) in our analysis. The MPK can be defined as the increase in productivity from one unit increase in capital:

$$MPK = \frac{df(k)}{dk} \tag{11}$$

When consumption is maximized, the MPK must equal the combined rates of depreciation and population growth $(\delta + n)$:

$$MPK = \delta + n \tag{12}$$

The Solow Growth Model underscores the positive correlation between capital per worker and long-term economic growth. Investment drives capital accumulation, with the savings rate determining the pace of capital growth. However, depreciation and population growth counteract this process. This culminates in economies gravitating toward a steady state of capital in the long run, where the savings rate equals the combined impact of population growth and depreciation.

In sum, the Solow Model elegantly captures the intricate dance between capital, savings, depreciation, and technological progress in shaping an economy's growth trajectory. It offers insights into how these factors interact and ultimately determine the pace of economic advancement. The next section will refine the model by introducing more accurate assumptions that align with the dynamics of an aging society.

## 3 Modifying the Model

First, we will introduce the new assumptions the augmented model is based on:

1. The overall population remains constant as opposed to growing by $n$.

2. The constant population N consists of young people $N_y$ and old people $N_o$ , $N = N_y + N_o$ . The proportion of old people is $p$ , $N_o = pN$ . Since $p$ is increasing, let $m = \Delta p > 0$.

3. All young people participate in the labor force, and a factor of $q$ old people are in the labor force. Labor force $L = N_y + qN_o = (1 - p)N + pqN$.

4. While much literature merely shows the correlation effects between aging and savings rate, Hu (2015) along with Pascual-Saez et al. suggest that there is a strong causal effect from the young population to the national savings rate. Then, one may assume the savings rate $s$ is the product of a natural savings tendency $t$, given as an exogenous variable and the percentage of the population that is young people $(1\text{-}p)$. $s = (1\text{-}p)t$. Then, $s$ decreases by $m$ each year.

Note that under these assumptions, output per worker is still a function of capital per worker, and the degree of importance of capital to economic growth, which is the core idea of the model, is still reflected.

## 3.1 The Augmented Consumption Function

As the original model, the total output is:

$$y = c + i \tag{13}$$

Assuming the population saves a fraction of $s$, where $s = pt$ ,one may express $c$ as

$$c = (1 - p)\,ty \tag{14}$$

Thus, $y$ and $i$ will be expressed as

$$y = (1 - pt)\,y + i \tag{15}$$

$$i = (1 - p)\,ty \tag{16}$$

This result has preserved the idea that investment is equal to savings and that savings rate is essential to capital formation which fuels economic growth. However, this work have modified the savings rate to reflect reality more accurately in an aging population. This will be important in later analysis of the steady state.

## 3.2 The Steady State in the Augmented Model

One can now express investments as a function of capital stock per worker

$$i = sf\,(k) \tag{17}$$

Now, including depreciation as a variable in the function, the annual difference in capital stock can be described as

$$\Delta k = \Delta sf\,(k) - \delta k \tag{18}$$

Observe that $s$ is now a variable that changes with time, therefore, to correctly reflect the change in $k$, one must also reflect the change in $s$.

$$\Delta k = (1 - (1 + m)\, p)\, t f\,(k) - \delta k \tag{19}$$

This work leads to the conclusion that the steady-state ($\Delta k = 0$), in this case, will not only depend on depreciation but also the distribution of the population. Since an aging population, $m$is constantly decreasing, one can conclude the steady state of the economy will constantly decrease as well, implying that the economic growth rate in an aging population will decrease with the degree that it is aged. Since investment(savings) is crucial to economic growth, the model suggests immediate policy changes to amend the negative impacts caused by aging.

## 3.3 The Steady State including Demographic Change

Another critical factor of capital per worker is the difference in demographic statistics every year. Since we assumed the overall population stays constant while the proportion of old and young people changes, ceteris paribus, capital per worker increases.

One can first express the annual decrease in the labor force L as:

$$\Delta L = \frac{[(1 - (1 + m)\, p)\, N + (1 + m)\, pqN]}{[(1 - p)\, N + pqN]} - 1 \tag{20}$$

Including the rate that the demographic build changes $m$ into the function, one can rewrite:

$$\Delta k = i - (\delta + \Delta L)\, k \tag{21}$$

Consequently, replacing $i$ for capital stock per-worker:

$$\Delta k = (1 - (1 + m)\, p)\, t f\,(k) - \left\{ \delta - 1 + \frac{[(1 - (1 + m)\, p)\, N + (1 + m)\, pqN]}{[(1 - p)\, N + pqN]} \right\} k \tag{22}$$

This expression depicts the change in capital as a function of demographic statistics, depreciation, and the population's natural saving tendency. While accounting for depreciation, the outflow of the labor force is also addressed. Thus, this work suggests a tool for analysis in the scenario of an aging population.

For example, we can now analyze the effects of different aging rates on $\Delta k$ . Taking the partial derivative with respect to $m$:

$$\frac{\partial \Delta k}{\partial m} = -\frac{p[f(k)\,pqt - f(k)\,(p-1)\,t + q - 1]}{p\,(q-1) + 1} \tag{23}$$

Using this we can determine the exact effects of changes in $m$ on $\Delta k$ given the circumstances of the geographical situation one concludes from statistics.

# 4 Conclusion

As the world continues to age, the academic world is bound to develop new tools that suit the economic analysis needs. This study first introduced the main implications and ideas behind the Solow Growth Model. With the key ideas defined, it then attempts to modify the Solow Growth Model to satisfy the need for better-suited models in a global aging trend. The augmented model is able to predict savings (which in the assumptions for the Solow Growth Model is equivalent to investment and thus economic growth) through demographic statistics. This is realized by adding and modifying original assumptions from the Solow Growth Model and deriving mathematically.

Based on the analysis, some general policy recommendations for aging populations could be concluded. A) The retirement age could be adjusted such that more people remain in the workforce longer. Adjusting retirement policies will directly change the definition of old and young people. This will have an exponential effect on the severity of aging on economic growth, since as the result expression reflects, the proportion of young and old people in the population is deterministic to investment levels and thus total output. B) Promoting and ensuring the effectiveness of retirement insurance products in the finance industry. An efficient finance sector in a society will be able to precisely allocate an individual's savings such that before the individual is old, the savings turn into investments and create value which in turn could be used to lower their retirement financial burden. C) The public sector could encourage the widespread of nursing homes, as it is much cheaper for society as a whole to take care of the old collectively as opposed to privately. Moreover, the public sector could even build official nursing homes, where they could offer jobs and training for the unemployed as caretaking is a low-threshold profession. This in turn contributes to the participation rate of the young people in the labor force.

This study contributes to literature mainly by exploring the topic of aged society growth models with an introductory model. However, we were still not able to include technology as an endogenous variable. Also, we assumed the overall population did not change, which is rarely the case in reality. Both of these amendments are potentially meaningful and could lead to a fruitful research topic for future researchers.

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